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But a single angle ACB can always be drawn less than two right angles and in such manner as that it shall differ from two right angles by as small an angle a as we please. The single angle ACB may, therefore, be made equal to any assumed sum whatever (less than two right angles) of the three angles of a rectilineal triangle.

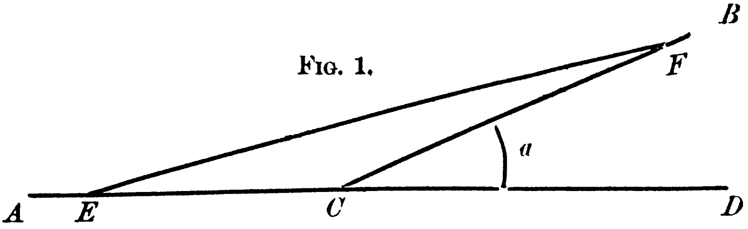

FIG. 1.

Let the angle ACB, Fig. 1, be equal to the assumed angle-sum of a rectilineal triangle, to wit: two right angles—a. Then, a straight line can not be drawn from any point E on the leg AC to any point F on the leg CB;for if that is granted, we shall have a triangle ECF whose angle-sum is greater than two right angles—a, which is against the hypothesis. But to refuse to grant that a straight line may be drawn from any point E to any other point F is to discredit postulate 1, of Euclid's Elements. The assumptions of Lobatschewsky's geometry are at war with Euclid's 1st postulate.

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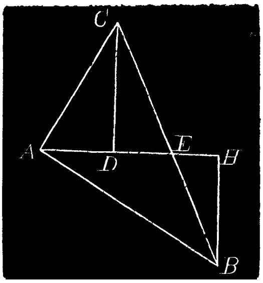
DEMONSTRATION OF THE "TANGENT PROPOSITION," AND OTHERS.

By Professor P. H. PHILBRICK, M. S., C. E., Lake Charles, Louisiana.

Let ABC be the triangle considered. Represent the parts as usual. Make $CE = CA$ and draw CD and BH perpendicular to AE and AE prolonged.

We have $A = CAE + BAE$; and $B = AEC - BAE = CAE - BAE$. Adding and dividing by two,gives $CAE = \frac{1}{2}(A+B)$. Subtracting and dividing by two gives $BAE = \frac{1}{2}(A-B)$.



The triangles ACD, ECD, and BEH are similar.

Hence, $\dfrac{AD}{EH} = \dfrac{AC}{BE}$, $\dfrac{DE}{EH} = \dfrac{CE}{BE}$, and $\dfrac{EH}{EH} = \dfrac{BE}{BE}$.

Adding, we have $\dfrac{AD+DE+EH}{EH} = \dfrac{AC+CE+EB}{BE}$.

That is, $\dfrac{AH}{EH} = \dfrac{a+b}{a-b}$ or $\dfrac{EH}{AH} = \dfrac{a-b}{a+b}$.

Also, $\dfrac{BH}{AH} = \tan\frac{1}{2}(A-B)$ and $\dfrac{BH}{EH} = \tan\frac{1}{2}(A+B)$.

Dividing gives $\dfrac{EH}{AH}=\dfrac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}$.

Equating, we have $\dfrac{a-b}{a+b}=\dfrac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}$......(1).

From triangle BAE, $\dfrac{BE}{AB}=\dfrac{\sin BAE}{\sin AEC}$, or $\dfrac{a-b}{c}=\dfrac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}$...(2)

Also $AH=AD+DE+EH=AC\cos\frac{1}{2}(A+B)+CE\cos\frac{1}{2}(A+B)+EB\cos\frac{1}{2}(A+B)$, $=(a+b)\cos\frac{1}{2}(A+B)$.

In triangle BAH, $AH=c\cos\frac{1}{2}(A-B)$. Equating these values of AH gives $\dfrac{a+b}{c}=\dfrac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}$....(3) The quotient of (2) divided by (3) gives (1) also.

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CERTAIN SERIES OF INTEGRAL, RATIONAL, SCALENE TRIANGLES.

By SYLVESTER ROBINS, Long Branch Depot, New Jersey.

Investigation leads to the belief that there is an endless number of infinite series of integral, rational, scalene triangles, wherein the base of every term in nth series is $n+2$, and the difference between the other two sides is constant.

EXAMPLES:

I. 3, 4, 5: 3, 25, 26: 3, 148, 149: 3, 865, 866: 3, 5044, 5045: &c.
II. 4, 3, 5: 4, 13, 15: 4, 51, 53: 4, 191, 193: 4, 723, 725: &c.
III. 5, 3, 4: 5, 29, 30: 5, 291, 292: 5, 2885, 2886: 5, 28563, 28564: &c.
Also;5, 5, 6: 5, 51, 52: 5, 509, 510: 5, 5043, 5044: 5, 49925, 49926: &c.
And;5, 12, 13: 5, 122, 123: 5, 1212, 1213: 5, 12002, 12003: 5,118112,118113: &c.
IV. 6, 25, 29: 6, 481, 485: 6, 8665, 8669: 6, 155521, 155525: &c.
V. 7, 15, 20: 7, 169, 174: 7, 1695, 1700: 7, 16801, 16806: &c.
VI. 8, 15, 17: 8, 123, 125: 8, 975, 977: 8, 7683, 7685: 8, 60495, 60497: &c.
VII. 9, 10, 17: 9, 73, 80: 9, 442, 449: 9, 2593, 2600: 9, 15130, 15137: &c.
VIII. 10, 17, 21: 10, 35, 39: 10, 273, 277: 10, 4049, 4053: &c.
IX. 11, 25, 30: 11, 267, 272: 11, 2665, 2670: 11, 26403, 26408: &c.
X. 12, 17, 25: 12, 365, 373: 12, 6617, 6625: 12, 118805, 118813: &c.
XI. 13, 20, 21: 13, 518, 519: 13, 13460, 13461: 13, 349454, 349455: &c.
XII. 14, 61, 65: 14, 1125, 1129: 14, 20221, 20225: 14, 362885, 362889: &c.
XIII. 15, 7, 20: 15, 28, 41: 15, 106, 119: 15, 1015, 1028: &c.
XIV. 16, 25, 39: 16, 241, 255: 16, 1945, 1959: 16, 15361, 15375: &c.
XV. 17, 25, 26: 17, 144, 145: 17, 841, 842: 17, 4904,4905: 17,28585,28586: &c.
XVI.
XVII. 19, 20, 37: 19, 153, 170: 19, 932, 949: 19,5473, 5490: 19,31940, 31957: &c.
XVIII. 20, 53, 55: 20, 1067, 1069: 20, 21305, 21307: 20, 425051, 425053: &c.
XIX. 21, 13, 20: 21, 82, 89: 21, 493, 500: 21, 2890, 2897: 21, 16861, 16868: &c.
XX.
XXI. 23, 123, 130: 23, 2768, 2775: 23, 60843, 60850: 23, 1336848, 1336855: &c